



PHYSICS ACADEMY

CAREER SPECTRA







Institute for IIT-JAM | CSIR-NET/JRF | U-SET | GATE | JEST | TIFR | BARC

Dedicated to excellence...

IIT-JAM

(MATHEMATICAL METHODS)

PREVIOUS YEAR'S QUESTIONS WITH ANSWER
(CHAPTER-WISE)

-  **MATRIX ALGEBRA**
-  **VECTOR ANALYSIS**
-  **FOURIER SERIES**
-  **ALGEBRA OF COMPLEX NUMBERS**
-  **DIFFERENTIAL EQUATIONS**
-  **OTHER QUESTIONS**

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MATRIX ALGEBRA

- Which of the following is **INCORRECT** for the matrix $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. **[IIT-JAM 2005]**
 - It is its own inverse
 - It is its own transpose
 - It is non-orthogonal
 - It has eigen values ± 1

- The symmetric part of $P = \begin{pmatrix} a \\ b \end{pmatrix} (a - 2b)$ is. **[IIT-JAM 2006]**
 - $\begin{pmatrix} a^2 - 2 & ba - 1 \\ ba - 1 & b^2 - 2 \end{pmatrix}$
 - $\begin{pmatrix} a(a - 2) & b \\ b & b^2 \end{pmatrix}$
 - $\begin{pmatrix} a(a - 1) & b(a - 1) \\ b(a - 1) & b^2 \end{pmatrix}$
 - $\begin{pmatrix} a(a - 2) & b(a - 1) \\ b(a - 1) & b^2 \end{pmatrix}$

- $(x \ y) \begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 15$. **[IIT-JAM 2007]**
 The matrix equation of above represents
 - a circle of radius $\sqrt{15}$
 - an ellipse of semi major axis $\sqrt{5}$
 - an ellipse of semi major axis 5
 - a hyperbola

- The product PQ of any two real, symmetric matrices P and Q is. **[IIT-JAM 2008]**
 - Symmetric for all P and Q
 - Never symmetric
 - Symmetric, if $PQ = QP$
 - Anti-symmetric for all P and Q

- A matrix is given by $M = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$. The eigenvalues of M are. **[IIT-JAM 2010]**
 - Real and positive
 - Purely imaginary with modulus 1
 - Complex with modulus 1
 - Real and negative

- Given two $(n \times n)$ matrices \hat{P} and \hat{Q} is Hermitian and \hat{Q} is skew (anti)-Hermitian. Which one of the following combinations of \hat{P} and \hat{Q} is necessarily a Hermitian matrix? **[IIT-JAM 2011]**
 - $\hat{P}\hat{Q}$
 - $i\hat{P}\hat{Q}$
 - $\hat{P} + i\hat{Q}$
 - $\hat{P} + \hat{Q}$

7. The inverse of the matrix $M = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ is. **[IIT-JAM 2013]**

- (a) $M - I$ (b) $M^2 - I$ (c) $I - M^2$ (d) $I - M$
Where I is the identity matrix.

8. The trace of a 2×2 matrix is 4 and its determinant is 8. If one of the eigenvalues is $2(1+e)$, the other eigenvalue is. **[IIT-JAM 2015]**

- (a) $2(1 - i)$ (b) $2(1 + i)$
(c) $(1 + 2i)$ (d) $(1 - 2i)$

9. The eigenvalues of the matrix representing the following pair of linear equations

$$\begin{aligned} x + iy &= 0 \\ ix + y &= 0 \end{aligned}$$

are **[IIT-JAM 2016]**

- (a) $1 + i, 1 + I$ (b) $1 - i, 1 - i$ (c) $1, i$ (d) $1 - i, 1 - i$

10. For the three matrices given below, which one of the choices is correct? $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, **[IIT-JAM 2017]**

- (a) $\sigma_1 \sigma_2 = -i \sigma_3$ (b) $\sigma_1 \sigma_2 = i \sigma_3$
(c) $\sigma_1 \sigma_2 + \sigma_2 \sigma_1 = I$ (d) $\sigma_3 \sigma_2 = -i \sigma_1$

11. Let matrix $M = \begin{pmatrix} 4 & x \\ 6 & 9 \end{pmatrix}$. If $\det(M)=0$, then **[IIT-JAM 2018]**

- (a) M is symmetric (b) M is invertible
(c) one eigenvalue is 13 (d) Its eigenvectors are orthogonal

12. The eigenvalues of $\begin{pmatrix} 3 & i & 0 \\ -i & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ are. **[IIT-JAM 2019]**

- (a) 2,4 and 6 (b) $2i, 4i$ and 6
(c) $2i, 4$ and 8 (d) 0,4 and 8

VECTOR ANALYSIS

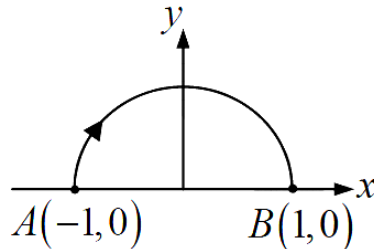
13. The equation of a surface of revolution is $z = \pm \sqrt{\frac{3}{2}x^2 + \frac{3}{2}y^2}$. The unit vector normal to the surface at the point $A \left(\sqrt{\frac{2}{3}}, 0, 1 \right)$ is. **[IIT-JAM 2010]**

- (a) $\sqrt{\frac{3}{5}}\hat{i} + \frac{2}{\sqrt{10}}\hat{k}$ (b) $\sqrt{\frac{3}{5}}\hat{i} - \frac{2}{\sqrt{10}}\hat{k}$

(c) $\sqrt{\frac{3}{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{k}$

(d) $\sqrt{\frac{3}{10}}\hat{i} + \frac{2}{\sqrt{10}}\hat{k}$

14. The line integral $\int_A^B \vec{F} \cdot d\vec{l}$, where $\vec{F} = \frac{x}{\sqrt{x^2+y^2}}\hat{x} + \frac{y}{\sqrt{x^2+y^2}}\hat{y}$, along the semi-circular path as shown in the figure below is. **[IIT-JAM 2011]**



- (a) -2 (b) 0 (c) 2 (d) 4

15. If \vec{F} is a constant vector and \vec{r} is the position vector then $\vec{\nabla}(\vec{F} \cdot \vec{r})$ would be. **[IIT-JAM 2012]**

- (a) $(\vec{\nabla} \cdot \vec{r})\vec{F}$ (b) \vec{F} (c) $(\vec{\nabla} \cdot \vec{F})\vec{r}$ (d) $|\vec{r}|\vec{F}$

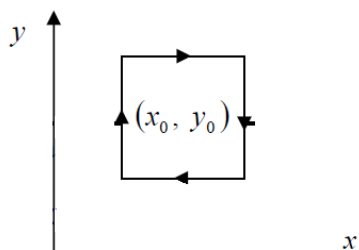
16. For vectors $\vec{a} = \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{c} = \hat{j} - \hat{k}$, the vector product $\vec{a} \times (\vec{b} \times \vec{c})$ is. **[IIT-JAM 2014]**

- (a) In the same direction as \vec{c} (b) In the direction opposite to \vec{c}
 (c) In the same direction as \vec{b} (d) In the direction opposite to \vec{b}

17. If the surface integral of the field $\vec{A}(x, y, z) = 2\alpha x\hat{i} + \beta y\hat{j} - 3\gamma z\hat{k}$ over the closed surface of an arbitrary unit sphere is to be zero, then the relationship between α , β and γ is. **[IIT-JAM 2014]**

- (a) $\alpha + \beta/6 - \gamma = 0$ (b) $\alpha/3 + \beta/6 - \gamma/2 = 0$
 (c) $\alpha/2 + \beta - \gamma/3 = 0$ (d) $2/\alpha + 1/\beta - 3/\gamma = 0$

18. The line integral $\oint \vec{A} \cdot d\vec{l}$ of a vector field $\vec{A}(x, y) = \frac{1}{r^2}(-y\hat{i} + x\hat{j})$ where $r^2 = x^2 + y^2$ is taken around a square (see figure) of side of unit length and centered at (x_0, y_0) with $|x_0| > \frac{1}{2}$ and $|y_0| > \frac{1}{2}$. If the value of the integral is L , then **[IIT-JAM 2014]**



- (a) L depends on (x_0, y_0)
 (b) L is independent of (x_0, y_0) and its value is -1

- (c) L is independent of (x_0, y_0) and its value is 0
- (d) L is independent of (x_0, y_0) and its value is 2

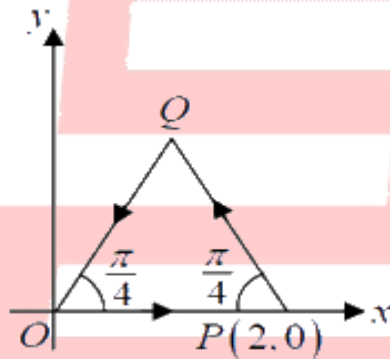
19. Consider a vector field $\vec{F} = y\hat{i} + xz^3\hat{j} - zy\hat{k}$. Let C be the circle $x^2 + y^2 = 4$ on the plane $z = 2$, oriented counter-clockwise. The value of the contour integral $\oint_C \vec{F} \cdot d\vec{r}$ is. **[IIT-JAM 2015]**

(a) 28π (b) 4π (c) -4π (d) -28π

20. The tangent line to the curve $x^2 + xy + 5 = 0$ at $(1,1)$ is represented by. **[IIT-JAM 2016]**

(a) $y = 3x - 2$ (b) $y = -3x + 4$
 (c) $x = 3y - 2$ (d) $x = -3y + 4$

21. Consider a closed triangular contour traversed in counter-clockwise direction, as shown in the figure. The value of the integral $\oint \vec{F} \cdot d\vec{l}$ evaluated along this contour, for a vector field, $\vec{F} = y\hat{e}_x - x\hat{e}_y$, is (\hat{e}_x, \hat{e}_y and \hat{e}_z are unit vectors in Cartesian-coordinate system). **[IIT-JAM 2016]**



22. A hemispherical shell is placed on the x-y plane centered at the origin. For a vector Field $\vec{E} = (-y\hat{e}_x + x\hat{e}_y)/(x^2 + y^2)$, the value of the integral $\int_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{a}$ over the hemispherical surface is..... π . ($d\vec{a}$ is the elemental surface area, \hat{e}_x, \hat{e}_y and \hat{e}_z are unit vectors in Cartesian-coordinate system). **[IIT-JAM 2016]**

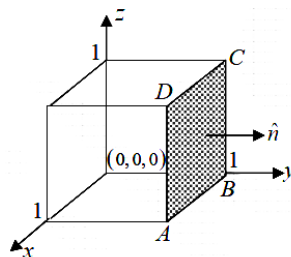
23. The integral of the vector $\vec{A}(\rho, \phi, z) = \frac{40}{\rho} \cos \phi \hat{\rho}$ (standard notation for cylindrical coordinates is used) over the volume of a cylinder of height L and radius R_0 is: **[IIT-JAM 2017]**

(a) $20\pi R_0 L(\hat{i} + \hat{j})$ (b) 0
 (c) $40\pi R_0 L\hat{j}$ (d) $40\pi R_0 L\hat{i}$

24. The volume integral of the function $f(r, \theta, \phi) = r^2 \cos \theta$ over the region $(0 \leq r \leq 2, 0 \leq \theta \leq \frac{\pi}{3}$ and $0 \leq \phi \leq 2\pi)$ is..... (Specify your answer upto two digits after the decimal point) **[IIT-JAM 2017]**



25. Let $f(x, y) = x^3 - 2y^3$. The curve along which $\nabla^2 f = 0$ is. **[IIT-JAM 2018]**
 (a) $x = \sqrt{2}y$ (b) $x = 2y$
 (c) $x = \sqrt{6}y$ (d) $x = \frac{-y}{2}$
26. A curve is given by $\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$. The unit vector of the tangent to the curve at $t=1$ is. **[IIT-JAM 2018]**
 (a) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (b) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$
 (c) $\frac{\hat{i}+2\hat{j}+2\hat{k}}{3}$ (d) $\frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$
27. If $\phi(x, y, z)$ is a scalar function which satisfies the Laplace equation, then the gradient of ϕ is. **[IIT-JAM 2019]**
 (a) Solenoidal and irrotational (b) Solenoidal but not irrotational
 (c) Irrotational but not solenoidal (d) Neither Solenoidal nor irrotational
28. A unit vector perpendicular to the plane containing $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} + \hat{k}$ is. **[IIT-JAM 2019]**
 (a) $\frac{1}{\sqrt{26}}(-\hat{i} + 3\hat{j} - 4\hat{k})$ (b) $\frac{1}{\sqrt{19}}(-\hat{i} + 3\hat{j} - 3\hat{k})$
 (c) $\frac{1}{\sqrt{35}}(-\hat{i} + 5\hat{j} - 3\hat{k})$ (d) $\frac{1}{\sqrt{26}}(-\hat{i} - \hat{j} - 3\hat{k})$
29. The gradient of scalar field $S(x, y, z)$ has the following characteristic(s). **[IIT-JAM 2019]**
 (a) Line integral of a gradient is path-independent
 (b) Closed line integral of a gradient is zero
 (c) Gradient of S is a measure of the maximum rate of change in the field S
 (d) Gradient of S is a scalar quantity
30. The flux of the function $\vec{F} = (y^2)\hat{x} + (3xy - z^2)\hat{y} + (4yz)\hat{z}$ passing through the surface ABC along \hat{n} is _____ (Round off to 2 decimal places). **[IIT-JAM 2019]**

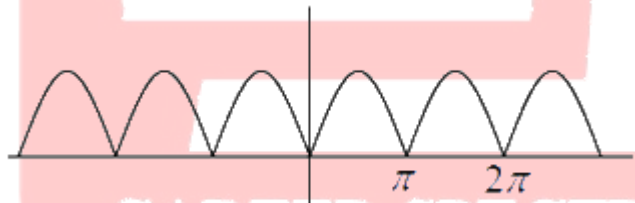


FOURIER SERIES

31. A periodic function can be expressed in a Fourier series of the form, $f(x) = \sum_{n=0}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$. The functions $f_1(x) = \cos^2 x$ and $f_2(x) = \sin^2 x$ are expanded in their respective Fourier series. If the coefficients for the first series are $a_n^{(1)}$ and $b_n^{(1)}$, and the coefficients for the second series are $a_n^{(2)}$ and $b_n^{(2)}$, then which of the following is correct? **[IIT-JAM 2005]**
- (a) $a_n^{(1)} = \frac{1}{2}$ and $b_n^{(2)} = \frac{-1}{2}$ (b) $b_2^{(1)} = \frac{1}{2}$ and $a_2^{(2)} = \frac{-1}{2}$
 (c) $a_2^{(1)} = \frac{1}{2}$ and $a_2^{(2)} = \frac{-1}{2}$ (b) $b_2^{(1)} = \frac{1}{2}$ and $b_2^{(2)} = \frac{-1}{2}$

32. $f(x)$ is a periodic function of x with a period of 2π . In the interval $-\pi < x < \pi$, $f(x)$ is given by $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$ In the expansion of $f(x)$ as a Fourier series of sine and cosine functions, the coefficient of $\cos(2x)$ is. **[IIT-JAM 2007]**
- (a) $\frac{2}{3\pi}$ (b) $\frac{1}{\pi}$ (c) 0 (a) $-\frac{2}{3\pi}$

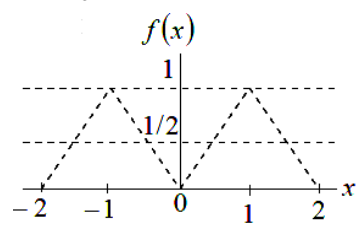
33. In the Fourier series of the periodic function (shown in the figure) $f(x) = |\sin x| = \sum_{n=0}^{\infty} [\alpha_n \cos nx + \beta_n \sin nx]$. Which of the following coefficients are non-zero? **[IIT-JAM 2009]**



- (a) α_n for odd n (b) α_n for even n
 (c) β_n for odd n (d) β_n for even n

34. Given that $f(1) = 1, f'(1) = 1$, and $f''(1) = 1$, the value of $f(1/2)$ is **[IIT-JAM 2013]**
 Ans.: 0.606

35. The Fourier series for an arbitrary periodic function with period $2L$, is given by $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$. For the particular periodic function shown in the figure the value of a_0 is. **[IIT-JAM 2015]**





- (a) 0 (b) 0.5 (c) 1 (d) 2

36. Fourier series of a given function $f(x)$ in the interval 0 to L is. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nx}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nx}{L}\right)$. If $f(x) = x$ in the region $(0, \pi)$, b_2

..... **[IIT-JAM 2016]**

37. For the Fourier series of the following function of period 2π . $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$

[IIT-JAM 2017]

the ratio (to the nearest integer) of the Fourier coefficients of the first and the third harmonic is:

- (a) 1 (b) 2 (c) 3 (d) 6

38. The function $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$ is expanded as a Fourier series of the form $a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$. Which of the following is true?

[IIT-JAM 2018]

- (a) $a_0 \neq 0, b_n = 0$ (b) $a_0 \neq 0, b_n \neq 0$
 (c) $a_0 = 0, b_n = 0$ (d) $a_0 = 0, b_n \neq 0$

ALGEBRA OF COMPLEX NUMBERS

39. The value of $\sqrt{i} + \sqrt{-i}$, where $i = \sqrt{-1}$, is

[IIT-JAM 2013]

- (a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

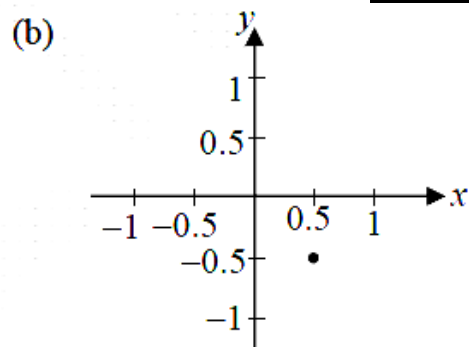
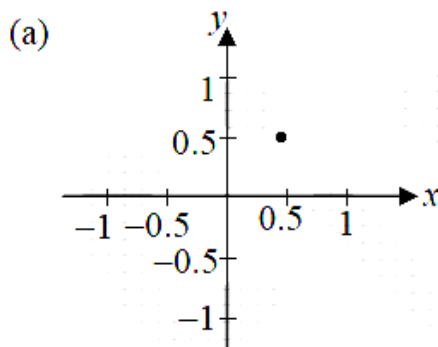
40. The phase of the complex number $(1+i)i$ in the polar representation is.

[IIT-JAM 2015]

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) $\frac{5\pi}{2}$

41. Which of the following points represent the complex number $= \frac{1}{1-i}$?

[IIT-JAM 2016]



[IIT-JAM 2015]

- (a) 2 (b) 1 (c) -1 (d) - 2

49. For the given set of equations $x + y = 1$, $y + z = 1$, $x + z = 1$, which one of the following statements is correct? [IIT-JAM 2016]

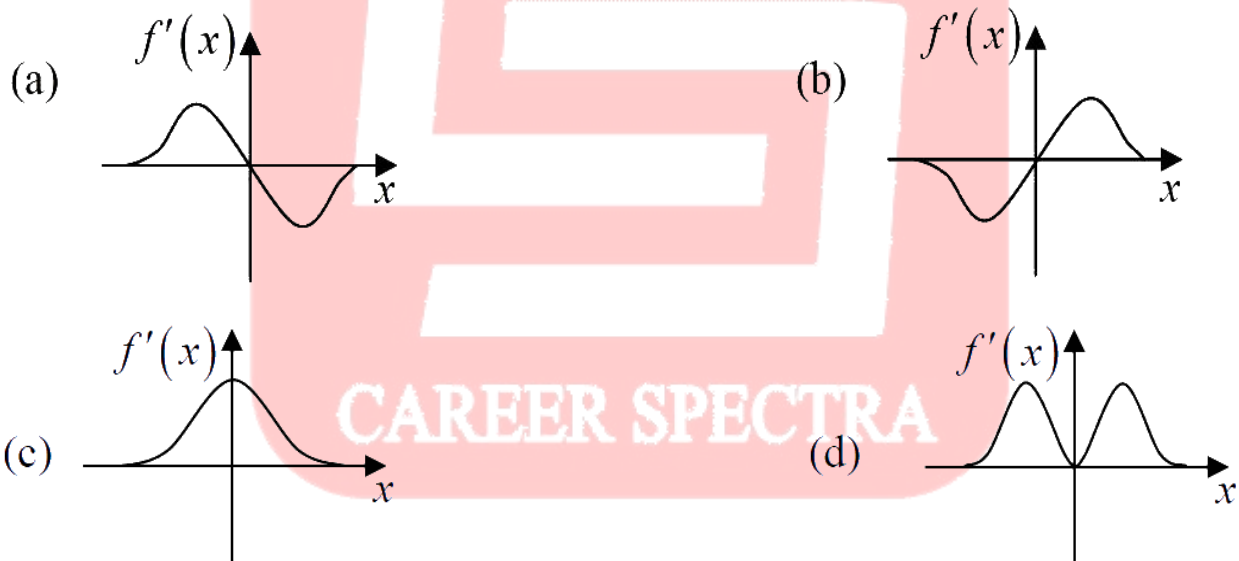
- (a) Equations are inconsistent
- (b) Equations are consistent and a single non-trivial solution exists
- (c) Equations are consistent and many solutions exist
- (d) Equations are consistent and only a trivial solution exists.

50. Consider a function $f(x, y) = x^3 + y^3$, where y represents a parabolic curve $x^2 + 1$. The total derivative of f with respect to x , at $x = 1$ is..... [IIT-JAM 2016]

[IIT-JAM 2016]

51. Which one of the following graphs represents the derivative $f'(x) = \frac{df}{dx}$ of the function $f(x) = \frac{1}{1+x^2}$ most closely (graphs are schematic and not drawn to scale)? [IIT-JAM 2017]

[IIT-JAM 2017]

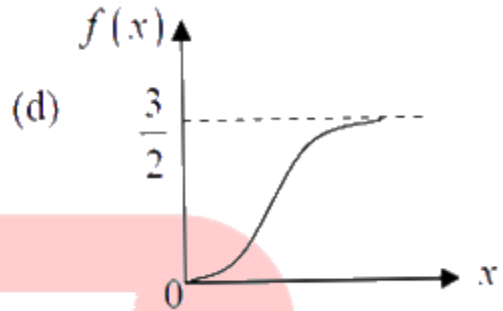
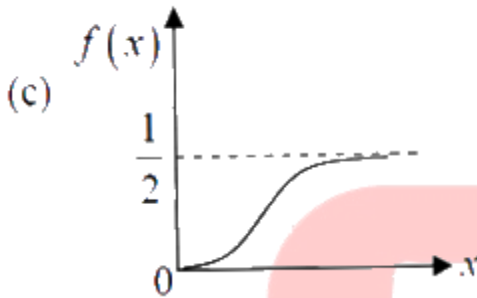
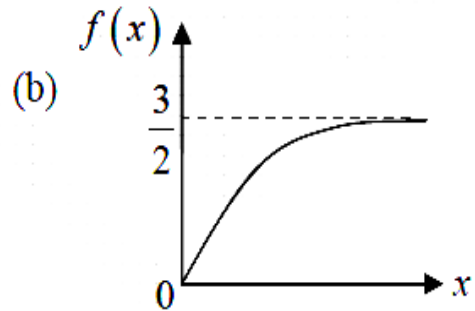
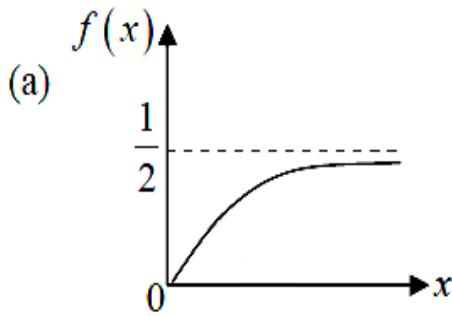


52. Consider two particles moving along the x - axis. In terms of their coordinates x_1 and x_2 , their velocities are given as $\frac{dx_1}{dt} = x_2 - x_1$ and $\frac{dx_2}{dt} = x_1 - x_2$, respectively. When they start moving from their initial locations of $x_1(0) = 1$ and $x_2(0) = -1$, the time dependence of both x_1 and x_2 contains a term of the form ea^t , where a is a constant. The value of a (an integer) is..... [IIT-JAM 2017]

[IIT-JAM 2017]

53. Which one of the following curves correctly represents (schematically) the solution for the equation $\frac{df}{dx} + 2f = 3$: $f(0) = 0$? [IIT-JAM 2018]

[IIT-JAM 2018]



54. Consider the transformation to a new set of coordinates (ξ, η) from rectangular Cartesian Coordinates (x, y) , where $\xi = 2x + 3y$ and $\eta = 3x - 2y$. In the (ξ, η) coordinate system, the area element $dx dy$ is. **[IIT-JAM 2018]**
- (a) $\frac{1}{13} d\xi d\eta$ (b) $\frac{2}{13} d\xi d\eta$ (c) $5 d\xi d\eta$ (d) $\frac{3}{5} d\xi d\eta$
55. Let $f(x) = 3x^6 - 2x^2 - 8$. Which of the following statements is (are) true? **[IIT-JAM 2018]**
- (a) The sum of all its roots is zero
 (b) The product of its roots is $-\frac{8}{3}$
 (c) The sum of all its roots is $\frac{2}{3}$
 (d) Complex roots are conjugates of each other.
56. The function $f(x) = \frac{8x}{x^2+9}$ is continuous everywhere except at. **[IIT-JAM 2019]**
- (a) $x = 0$ (b) $x = \pm 9$ (c) $x = \pm 9i$ (d) $x = \pm 3i$
57. The value of $\left| \int_0^{3+i} (\bar{z})^2 dz \right|^2$, along the line $3y = x$, where $z = x + iy$ is _____.
 (Round off to 1 decimal places). **[IIT-JAM 2019]**



ANSWER KEY

1.	C	2.	D	3.	B	4.	C	5.	C	6.	C
7.	B	8.	A	9.	D	10.	B	11.	A,C,D	12.	A
13.	B	14.	B	15.	B	16.	A	17.	B	18.	C
19.	A	20.	B	21.	-2	22.	2	23.	D	24.	15.07
25.	B	26.	D	27.	A	28.	D	29.	A,B,C	30.	1.17
31.	C	32.	D	33.	B	34.	0.606	35.	C	36.	0.5
37.	C	38.	B	39.	A	40.	C	41.	A	42.	0.33
43.	$Ce^{-(x^2+y^2)/2}$	44.	D	45.	0.27	46.	A	47.	A	48.	C
49.	B	50.	27	51.	A	52.	2	53.	B	54.	A
55.	A,B,D	56.	D	57.	111.1						